A2
Sem 2 review

1. $(k^3+4k^2-14k-5) \div (k-2)$

$$k^2+6k-2 \quad \text{R } -9$$

$$k^2+6k-2-\frac{9}{k-2}$$

$$-(k^3-2k^2)$$
$$6k^2-14k$$
$$-(6k^2-12k)$$
$$-2k-5$$
$$-(-2k+4)$$
$$-9$$

2. $(p^3-5p^2+7p-3) \div (p^2+0p-4)$

$$p-5 \quad \text{R } 11p-23$$

$$p-5+\frac{11p-23}{p^2-4}$$

$$-(p^3+0p^2-4p)$$
$$-5p^2+11p-3$$
$$-(-5p^2-0p+20)$$
$$11p-23$$

3. $b^3+4b^2-9 \div b+4$ $\quad b=-4$

-4	1	4	0	-9
	↓	-4	0	0
	1	0	0	-9

↑ remainder

$$b^2-\frac{9}{b+4}$$

4. $n^4-55n^2+36n+49 \div n-7$

7	1	0	-55	36	49
	↓	7	49	-42	-42
	1	7	-6	-6	7

$$n^3+7n^2-6n-6+\frac{7}{n-7}$$

5. $b^3+5b^2-26b-16 \div b+8$

-8	1	5	-26	-16
	↓	-8	24	16
	1	-3	-2	0

yes! remainder is 0.

6. $x^3 1$ 3 because degree is 3.

7. $x^4-x^2-20=0$ 4 because degree 4

9. $f(x) = 2x^3 + x^2 - 20x - 25$

L: $\pm 1, 5, 25$
P: $\pm 1, 2$
$\frac{L}{F}$: $\pm 1, \frac{1}{2}, 5, \frac{5}{2}, 25, \frac{25}{2}$

8. $f(x) = 9x^3 + 24x^2 + 12x - 7$
Last $\pm 1, \pm 7$
First $\pm 1, 3, 9$

$\frac{L}{F}$: $\pm 1, \frac{1}{3}, \frac{1}{9}, 7, \frac{7}{3}, \frac{7}{9}$

10. $2 + \sqrt{10}$, $2 + 3i$

$(x - 2 - \sqrt{10})(x - 2 + \sqrt{10})$ $(x - 2 - 3i)(x - 2 + 3i)$

$x = 2 - \sqrt{10}$ $x = 2 - 3i$

conjugates of both

11. $3 + 2\sqrt{2}, -i$
$3 - 2\sqrt{2}, i$ conjugates.

12. $y = (x-5)(x+1+2i)(x+1-2i)$ ← mult. these First

Don't need

$x^2 + x - 2ix + x + 1 - 2i$
$+ 2ix + 2i - 4i^2$
$x^2 + 2x + 1 - 4i^2 = x^2 + 2x + 5$

→ $(x-5)(x^2 + 2x + 5)$
$x^3 + 2x^2 + 5x - 5x^2 - 10x - 25$

$y = x^3 - 3x^2 - 5x - 25$

13. $y = x(x-3)(x+5)$

14. $y = (x-4)(x-\sqrt{10})(x+\sqrt{10})$ ← FOIL these First

$x^2 - \sqrt{10}x + \sqrt{10}x - \sqrt{100}$
$x^2 - 10$

$(x-4)(x^2-10)$ FOIL

$y = x^3 - 4x^2 - 10x + 40$

15. $(x-5)(x-2i)(x+2i)$

$(x-5)(x^2 - 2ix + 2ix - 4i^2)$

$(x-5)(x^2+4)$

$\boxed{y = x^3 - 5x^2 + 4x - 20}$

16. $y = x^3 - 2x^2 + 2x - 4$ poss: $\pm 1, 2, 4$

2 | 1 | -2 | 2 | -4

↓ | | 2 | 0 | 4

1 | 0 | 2 | 0

$\boxed{x = 2}$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm\sqrt{2} = \boxed{\pm i\sqrt{2}}$

17. $y = x^3 + x^2 - 5x - 5$

$\pm 1, \pm 5$

-1 | 1 | 1 | -5 | -5

↓ | | -1 | 0 | 5

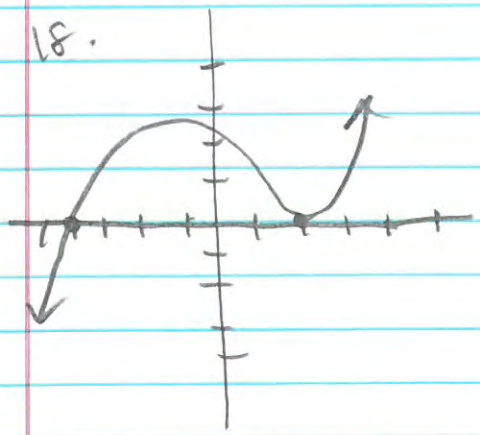
1 | 0 | -5 | 0

$\boxed{x = -1}$

$x^2 - 5 = 0$ $x^2 = 5$

$\boxed{x = \pm\sqrt{5}}$

18.



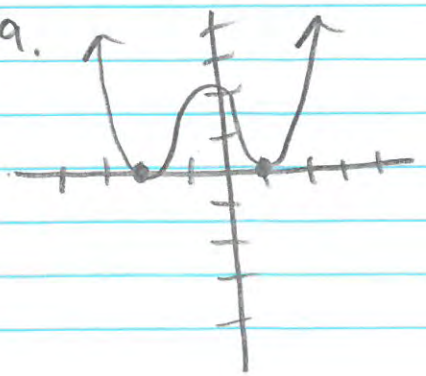
$y = (x-2)^2(x+4)$ ↙ ↗ x3

$x = 2$ $x = -4$

mult of 2 mult of 1

bounces

19.



$y = (x-1)^2(x+2)^2$

$x = 1$ $x = -2$

both mult. of 2

bounce

20. $x = -4$ mult of 2 $\quad x = 2$ mult of 1 $\quad (x+4)^2(x-2)$ degree 3

21. $y = x^3 + 4x^2 + 2x + 8$

$x = -4$

$-4 \mid 1 \quad 4 \quad 2 \quad 8$

$\quad\quad -4 \quad 0 \quad -8$

$1 \quad 0 \quad 2 \quad 0$

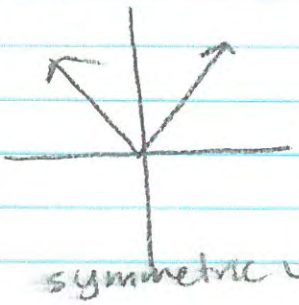
$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm\sqrt{-2}$

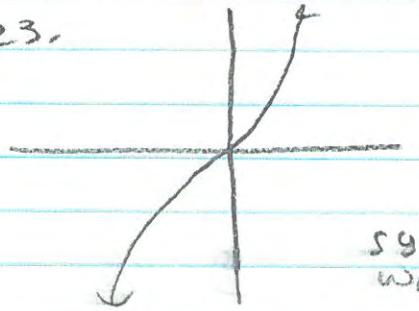
$\boxed{x = \pm i\sqrt{2}}$

22.



symmetric w/ y-axis

23.



symm. w/ origin

24. $f(x) = x^3 + x - 1$

$f(-x) = (-x)^3 + (-x) - 1$

$f(-x) = -x^3 - x - 1$

neither

all signs don't change (odd)

all signs don't stay (even)

25. $y = 2(x)^4 - 3x^2 - 12$

$f(-x) = 2(-x)^4 - 3(-x)^2 - 12$

$= 2x^4 - 3x^2 - 12$

even

all are same as original

26. $\sqrt[3]{54}$

$\sqrt[3]{27 \cdot 2}$

$\boxed{3\sqrt[3]{2}}$

27. $\sqrt{112x^4y^7}$

$\sqrt{16 \cdot 7\, x^2 x^2 y^2 y^2 y^2}$

$\boxed{4x^2y^2\sqrt{7}}$

28. $\dfrac{\sqrt[3]{48m^4n^3}}{\sqrt[3]{8 \cdot 6\, m^3 \cdot m\, n^3}}$

$\boxed{2mn\sqrt[3]{6m}}$

29. $-5\sqrt{32u^5v}$

$-5\sqrt{16 \cdot 2\, u^2 u^2 v}$

$\boxed{-20u^2\sqrt{2v}}$

$\sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$
$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

30. $3\sqrt{6} + 2\sqrt{6} + 2\sqrt{8}$
$\boxed{5\sqrt{6} + 2\sqrt{8}}$

31. $3\sqrt{2} - 3\sqrt{20} - 3\sqrt{18}$
$3\sqrt{2} - 3 \cdot 2\sqrt{5} - 3 \cdot 3\sqrt{2}$
$3\sqrt{2} - 6\sqrt{5} - 9\sqrt{2}$
$\boxed{-6\sqrt{2} - 6\sqrt{5}}$

32. $\sqrt{5}(5\sqrt{10} + 5)$
$5\sqrt{50} + 5\sqrt{5}$
$5 \cdot 5\sqrt{2} + 5\sqrt{5}$
$\boxed{25\sqrt{2} + 5\sqrt{5}}$

$\sqrt{50}$
$\sqrt{25 \cdot 2}$
$5\sqrt{2}$

33. $(\sqrt{3} + 5)(4\sqrt{3} - 3)$
$4\sqrt{9} - 3\sqrt{3} + 20\sqrt{3} - 15$
$12 + 17\sqrt{3} - 15$
$\boxed{-3 + 17\sqrt{3}}$

34. $\dfrac{2\sqrt{6}}{\sqrt{50}} = \dfrac{2\sqrt{6}}{5\sqrt{2}} = \boxed{\dfrac{2}{5}\sqrt{3}}$

35. $\dfrac{2}{5+3\sqrt{2}} \cdot \dfrac{5-3\sqrt{2}}{5-3\sqrt{2}} = \dfrac{10 - 6\sqrt{2}}{25 - 15\sqrt{2} + 15\sqrt{2} - 9\sqrt{4}} = \boxed{\dfrac{10-6\sqrt{2}}{7}}$

$-9 \cdot 2$
-18

36. $\dfrac{-4+3\sqrt{5}}{5+\sqrt{2}} \cdot \dfrac{5-\sqrt{2}}{5-\sqrt{2}} = \dfrac{-20 + 4\sqrt{2} + 15\sqrt{5} - 3\sqrt{10}}{25 - \sqrt{4}} =$
$\boxed{\dfrac{-20 + 4\sqrt{2} + 15\sqrt{5} - 3\sqrt{10}}{23}}$

37. $\sqrt{6x} = \boxed{(6x)^{1/2}}$

$1/2$ power is $\sqrt{}$
$1/3$ power is $\sqrt[3]{}$

38. $(\sqrt[4]{3x})^5 = \boxed{(3x)^{5/4}}$

39. $(4m)^{1/3} = \boxed{\sqrt[3]{4m}}$

40. $x^{-4/5} = (\sqrt[5]{x})^{-4} = \boxed{\dfrac{1}{(\sqrt[5]{x})^4}}$

41. $(x^6)^{1/2}$ mult. exp. $\boxed{x^3}$

42. $(64r^2)^{1/2} = 64^{1/2} \cdot r^3 = (\sqrt{64})^3 r^3 = \boxed{512r^3}$

43. $(25n^2)^{3/2} = 25^{3/2} n^3 = (\sqrt{25})^3 n^3 = \boxed{125n^3}$

44. $(100a^4)^{1/2} = 100^{1/2} a^2 = \boxed{10a^2}$

45. $2x^2 y^{5/3} \cdot 2x^2 y^4$ $\quad 5/3 + 4 = 5/3 + 12/3 = 17/3$

$\boxed{4x^4 y^{17/3}}$

46. $a^{3/2} b^{1/4} \cdot 3a \cdot 4ab^{2/3}$

$\boxed{12a^{7/2} b^{11/12}}$

$a \cdot a \cdot a^{3/2} =$
$1+1+3/2 = 7/2$

$b^{1/4} b^{2/3}$
$1/4 + 2/3 = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

47. $-1 = \sqrt{2x-6} - 3$

$+3 \quad +3$

$(2)^2 = (\sqrt{2x-6})^2$

$4 = 2x-6$

$10 = 2x$

$\boxed{5 = x}$

Check

$-1 = \sqrt{10-6} - 3$

$-1 = \sqrt{4} - 3$

$-1 = 2 - 3$ ✓

48. $(\sqrt{-8-m})^2 = (\sqrt{2m+22})^2$

$-8 - m = 2m + 22$

$-30 = 3m$

$\boxed{-10 = m}$

Check

$\sqrt{-8+10} = \sqrt{-20+22}$

$\sqrt{2} = \sqrt{2}$ ✓

49. $(\sqrt{7n-3})^2 = (n+1)^2$ FOIL

$7n-3 = n^2 + 2n + 1$

$0 = n^2 - 5n + 4$

$(n-4)(n-1)$

$\boxed{n=4 \quad n=1}$

-4×-1
-5

Check 4

$\sqrt{28-3} = 4+1$

$5 = 5$ ✓

Check 1

$\sqrt{4} = 2$

$2 = 2$ ✓

50. $r = \sqrt{2r-9} + 6$ Isolate $\sqrt{}$

$(r-6)^2 = (\sqrt{2r-9})^2$ square both / FOIL

$r^2 - 12r + 36 = 2r - 9$

$r^2 - 14r + 45 = 0$

45, -9, -5, -14

$(r-9)(r-5) = 0$

$r = 9 \quad r = 5$

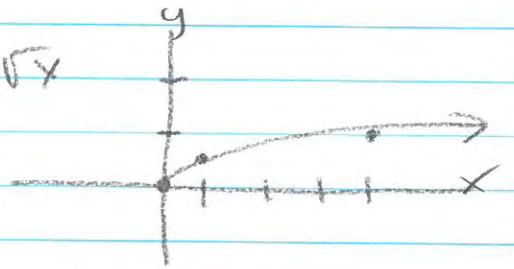
check 9: $9 = \sqrt{9} + 6$, $9 = 3 + 6$ ✓

check 5: $5 = \sqrt{1} + 6$, $5 \neq 1 + 6$

$\boxed{r = 9}$

51. $y = \frac{1}{2}\sqrt{x}$

x	y
0	0
1	½
4	1



$\sqrt{x}$ → half parabola sideways

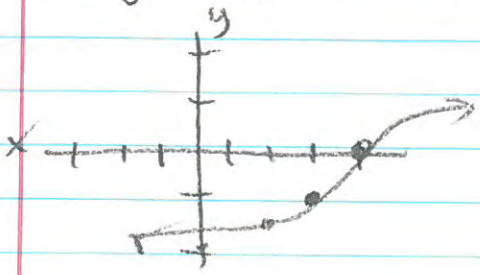
Domain $[0, \infty)$

Range $[0, \infty)$

52. $y = \sqrt[3]{x-3} - 1$

$\sqrt[3]{}$ r + 3 down 1

x	y
4	0
2	-2



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

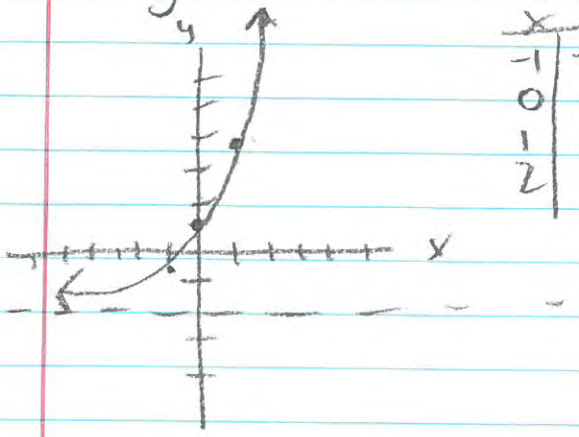
53. $y = 3 \cdot 2^x - 2$ down 2

x	y
-1	-1½
0	1
1	4
2	10

$3 \cdot 2^{-1} - 2 = \frac{3}{2} - 2 = -\frac{1}{2}$

$3 \cdot 2^0 - 2 = 1$

$6 - 2$

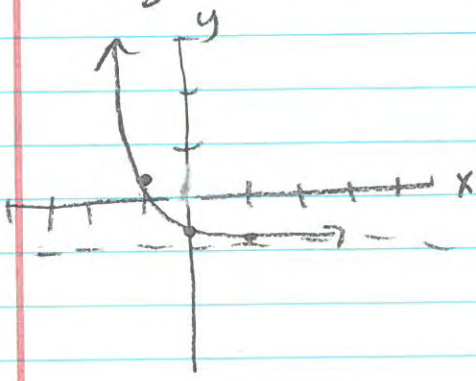


Domain $(-\infty, \infty)$

Range $(-2, \infty)$

decay

54. $y = \frac{1}{4}\left(\frac{1}{3}\right)^{x+1} - 1$ left 1 down 1



x	y
-1	1/4
0	-11/12
1	

$\frac{1}{12} - 1 = \frac{1}{12} - \frac{12}{12} = -11/12$

$\frac{1}{36} - 1 = -35/36$

D: $(-\infty, \infty)$

R: $(-1, \infty)$

55. $y = \log_3 (x-1) + 1$ $3^y = x - 1$

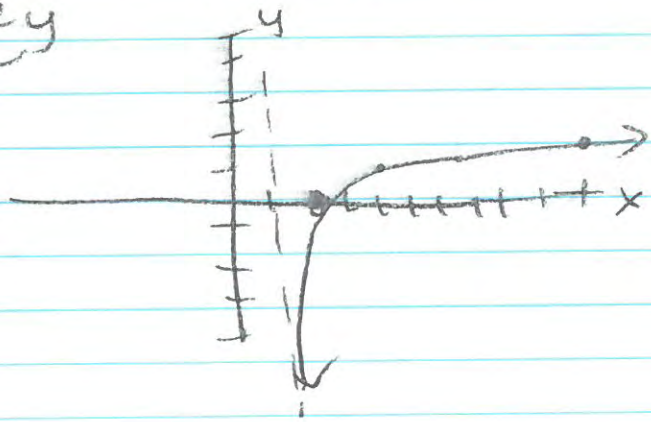
$3^{y+1} = x$

x	y
2	0
4	1
10	2

choose y

D: $(1, \infty)$

R: $(-\infty, \infty)$



Factor $x^3-2x^2+9x+18$

$x^2(x-2)+9(x-2)$

$(x^2-9)(x-2)$

$\boxed{(x+3)(x-3)(x-2)}$

(56) $f(x)-g(x) = 16-x^2-(4-x) = 16-x^2-4+x = \boxed{-x^2+x+12}$

(57) $f(x)\cdot g(x) = (1-x^2)(1-x) = 1-x-x^2+x^3 = \boxed{x^3-x^2-x+1}$

(58) $s(r(2))$ $\quad r(2) = 2(2) = 4$

$r = 2x$ $\quad s(4) = 4^3-3 = 64-3 = 61$

$s = x^3-3$

(59) $r = x^2-3$ $\quad r(s(-1))$ $\quad s(-1) = -1-6 = -7$

$s = x^3-6$ $\quad r(-7) = (-7)^2-3 = 49-3 = 46$

(60) $f(x) = (x-3)^2$ $\quad \boxed{g(x) = \frac{-2}{x}}$

$f(g(x)) = \boxed{\left(\frac{-2}{x}-3\right)^2}$ plug g into f

(61) $g(f(x)) = \boxed{\frac{-2}{(x-3)^2}}$ plug f into g

(62) inverse of $(1,7)(2,5)(3,3)(4,1) \rightarrow \boxed{(7,1)(5,2)(3,3)(1,4)}$

switch $x \rightarrow y$

(63) inverse of $y = 5x-3$ switch x, y

$x = 5y-3$ solve for y

$x+3 = 5y$

$\boxed{\frac{x+3}{5} = y}$

(64) inverse of $y = 18x - 7$

$x = 18y - 7$

$\frac{x+7}{18} = y$

(65) $g(x) = \frac{1}{2}x - \frac{1}{3}$ $\quad f(x) = \frac{6x+2}{3}$

$x = \frac{1}{2}y - \frac{1}{3}$

$6x = 3y - 2$

$6x + 2 = 3y$

$\frac{6x+2}{3} = y$ ✓

$f(x) = \frac{3}{4}x^3 - 1$

$y = \frac{3}{4}x^3 - 1$

$x = \frac{3}{4}x^3 - 1$

$x + 1 = \frac{3}{4}x^3$

$4x + 4 = 3x^3$

$\frac{4x+4}{3} = x^3$

$\sqrt[3]{\frac{4x+4}{3}} = x$

(66) $A = 250(1 + .10)^{15} = \$1044.31$

(67) $y = 290(1 + .09)^5 = 446$

(68) $A = 1000\left(1 + \frac{.08}{2}\right)^{2 \cdot 8}$ $\quad A = P\left(1 + \frac{r}{n}\right)^{nt}$

$\$1872.98$

69 $A = 1000\left(1 + \frac{.06}{12}\right)^{12 \cdot 10} = \$1819.40$

(70) $y = (390)(1 - .11)^5 = 218$

(71) $y = 320{,}000\,(1 - .15)^7 = \$102{,}584.67$

(72) $7e^{19}/_{35e^{6}} = \boxed{\frac{1}{5}e^{13}}$

(73) $e^{1.35} \approx \boxed{3.857}$

(74) $A = 5500e^{.09 \cdot 6} \approx \boxed{\$9438.04}$

(75) $A = 6000e^{.05 \cdot 7} \approx \boxed{\$8514.41}$

(76) $\boxed{\text{increasing}}$ because rate is .017 $\boxed{1.7\%}$

$t=0$ is now $P(0) = 17200e^{.017 \cdot 0} = \boxed{17200}$ people

$P(20) = 17200e^{.017 \cdot 20} \approx \boxed{24165}$ people

(77) $\log_{243} 729 = 6/5$ $\boxed{243^{6/5} = 729}$

(78) $\log_2 16$ $2^x = 16$ $(x=4)$

(79) $\log_7 \frac{1}{49}$ $7^x = \frac{1}{49}$ $(x=-2)$

(80) $\ln e^{-4} = -4$

(81) $\frac{1}{2}\log_5 16 - 3\log_5 x + 4\log_5 y$ bring in exp.

$\log_5 16^{1/2} - \log_5 x^3 + \log_5 y^4 = \log_5 \frac{4}{x^3} + \log_5 y^4$

$= \boxed{\log_5 \frac{4y^4}{x^3}}$

(82) $\ln \frac{2x}{y^4} = \ln 2x - \ln y^4 = \boxed{\ln 2 + \ln x - 4\ln y}$

(83) $\frac{1}{5}\log_3 32 - 2\log_3 x + \frac{1}{2}\log_3 y = \log_3 32^{1/5} - \log_3 x^2 + \log_3 y^{1/2}$

$= \log_3 \frac{2}{x^2} + \log_3 \sqrt{y} = \boxed{\log_3 \frac{2\sqrt{y}}{x^2}}$

(84) $\log_4 7 = \frac{\log 7}{\log 4} \approx \boxed{1.404}$

(85) $\log_3 5 = \frac{\log 5}{\log 3} \approx \boxed{1.465}$

(86) $e^{-3x} = 7.1$ ln both

$\ln e^{-3x} = \ln 7.1$

$-3x = \ln 7.1$

$x = \frac{\ln 7.1}{-3} \approx \boxed{-0.653}$

(87) $6^{-.2x} - 3 = 7 \rightarrow 6^{-.2x} = 10$

isolate base

$\log 6^{-.2x} = \log 10$ log both

$-.2x \log 6 = 1$

$-.1556x = 1 \rightarrow x = \boxed{-6.425}$

(88) $\log_4 (x+3) = -2$ circle of life

$4^{-2} = x+3$

$\frac{1}{16} = x+3$

$x = -2\frac{15}{16}$ or $\boxed{-47/16}$

(89) $\log_4 (x+6) + \log_4 x = 2$ product rule

$\log_4 x^2 + 6x = 2$

$4^2 = x^2 + 6x$ circle of life

$0 = x^2 + 6x - 16$ $0 = (x-2)(x+8)$ $\boxed{x=2}$ $x = \cancel{-8}$ extr.

(90) $\ln(x+7) = \ln(3x-5)$ ln=ln drop lns

$x+7 = 3x-5$

$12 = 2x$

$\boxed{6 = x}$

(91) $\log_2(-x) + \log_2(x+12) = 5$ product rule circle of life

$\log_2 -x^2 - 12x = 5$

$2^5 = -x^2 - 12x$

$x^2 + 12x + 32 = 0$

$(x+4)(x+8) = 0$

$\boxed{x = -4 \quad x = -8}$

(92) $4 \times 4 \times 6$ Fun counting principal

$= \boxed{96}$

(93) $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{3628800}$

(94) ${}_5P_3 = \frac{5!}{2!} = \boxed{60}$

(95) $5 \times 4 \times 3 \times 2 \times 1$ or ${}_5P_5 = \boxed{120}$

(96) minimum (repetition) $\frac{9!}{4!2!}$ = total reps.

$\boxed{7560}$

(97) ${}_8C_1 \, {}_3C_1 = 8 \times 3 = 24$

98. $_{10}C_2 = 45$

99. $_7C_6 = 7$

100. $_{15}C_4 = 1365$

101. permu. $5P5 = 120$ $\boxed{\frac{1}{120}}$

102. (c) comb.

103. 18/22

104. 1/8

105. $P(4 \text{ or } 3) = 1/6 + 1/6 = 2/6 = 1/3$

106. combo's $P(4 \text{ or } <6) = \frac{{}_{13}C_4}{{}_{52}C_4} + \frac{{}_{20}C_4}{{}_{52}C_4} - \frac{{}_{5}C_4}{{}_{52}C_4}$ (4 hearts; 1,2,3,4,5 ×4)

$$\frac{715 + 4845 - 15}{270725} \qquad \frac{5545}{270725}$$

$\boxed{\approx 0.02}$

107. 7 red 3 white

a) $P(R \text{ or } Odd) = P(R) + P(O) - P(R \text{ and } O)$

$\frac{7}{10} + \frac{5}{10} - \frac{3}{10} = \frac{9}{10}$

b) $P(W \text{ or } even) = P(W) + P(E) - P(W \text{ and } E)$

$= \frac{3}{10} + \frac{5}{10} - \frac{1}{10} = \frac{7}{10}$

(108) 23 calc 29 Fr 12 both

$P(C \text{ or } F) = \frac{23}{100} + \frac{29}{100} - \frac{12}{100} = \frac{40}{100} = \boxed{\frac{2}{5}}$ or 40%

(109) $P(\text{dv } 7 \text{ and even}) = \boxed{\frac{1}{15}}$
14

(110) $\boxed{5/6}$

(111) $\frac{8}{24_3} \cdot \frac{6}{24_4} = \boxed{\frac{1}{12}}$ independent

$P(B) \cdot P(W)$

(112) 51 total Ind. $\frac{21}{51} \cdot \frac{21}{51} = 441/2601 = \boxed{\frac{49}{289}}$

$\approx 17\%$

(113) $\frac{30}{51} \cdot \frac{30}{51} = \frac{900}{2601} = \boxed{\frac{100}{289}} \approx 35\%$

(114) $P(G) \cdot P(Y) = \frac{21}{51} \cdot \frac{30}{51} + \frac{30}{51} \cdot \frac{21}{51}$
6y ovy 6

$\frac{630 + 630}{2601} = \boxed{\frac{140}{289}} \approx 48\%$

(115) 8 total $P(P) \cdot P(R)$ dep.

$5/8 \cdot 3/7 = \boxed{15/56}$

	Full	Mid	comp.	sub	
(11b) Am.	12	18	15	5	50 am
Jap	9	21	12	6	48 Jap
Eur	8	12	6	4	30 Eur
	Full 29	Mid 51	Com. 33	Sub 15	128

a) A: am B: comp.

$P(A) = 50/128$ $P(A|\text{comp}) = 15/33 = (5/11)$

c) $\frac{15}{128} \cdot \frac{14}{127} \cdot \frac{13}{126} = \frac{2730}{2048256} \approx$ (0.0013 or .13%)

b) no you have compact and american

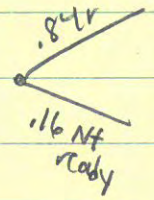
d) dependent. Once one comp American is sold, that reduces all.

(11.7) $P(H|M) = \frac{70}{180} = (\frac{7}{18}) \approx 0.389$

total males = 180

Hand m = 70

(11.8)



$P(\text{Work}|\text{ready}) = \frac{P(\text{w and R})}{P(\text{ready})} = \frac{.67}{.84} = .797$

or 79.8%